ΚΑΡΤΕΣΙΑΝΟ ΣΥΣΤΗΜΑ ΑΝΑΦΟΡΑΣ

$$\underline{r}(t) = x(t)\underline{e}_x + y(t)\underline{e}_y + z(t)\underline{e}_z$$

$$\underline{v} = \dot{\underline{r}} = \dot{x}\underline{e}_x + \dot{y}\underline{e}_y + \dot{z}\underline{e}_z$$

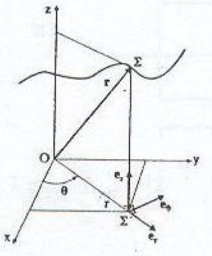
$$\underline{a} = \dot{\underline{v}} = \ddot{\underline{r}} = \ddot{x}\underline{e}_x + \ddot{y}\underline{e}_y + \ddot{z}\underline{e}_z$$

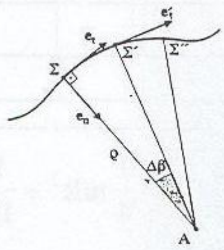
ΚΥΛΙΝΔΡΙΚΟ ΣΥΣΤΗΜΑ ΑΝΑΦΟΡΑΣ

$$\underline{r} = r\underline{e}_r + z\underline{e}_z, \quad \underline{v} = \dot{r}\underline{e}_r + r\dot{\theta}\underline{e}_\theta + \dot{z}\underline{e}_z$$

$$\underline{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta + \ddot{z}\underline{e}_z$$

$$\dot{\underline{e}}_r = \dot{\theta}\underline{e}_\theta, \quad \dot{\underline{e}}_\theta = -\dot{\theta}\underline{e}_r$$





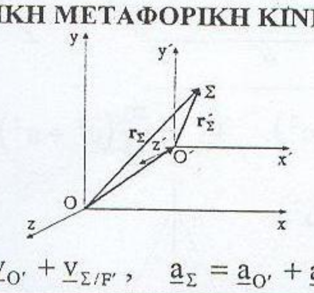
ΤΡΟΧΙΑΚΟ ΣΥΣΤΗΜΑ

$$\underline{r}(t) = \underline{r}(s(t)), \quad \underline{v} = \dot{s}\underline{e}_t$$

$$\underline{a} = \ddot{s}\underline{e}_t + \frac{\dot{s}^2}{\rho}\underline{e}_n$$

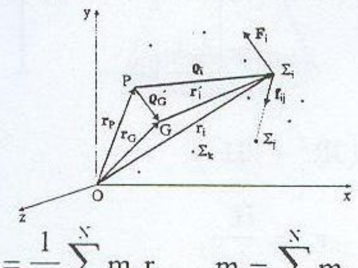
$$\underline{e}_t = \frac{d\underline{r}}{ds}, \quad \frac{d\underline{e}_t}{ds} = \frac{1}{\rho}\underline{e}_n$$

ΣΧΕΤΙΚΗ ΜΕΤΑΦΟΡΙΚΗ ΚΙΝΗΣΗ



$$\underline{v}_\Sigma = \underline{v}_{O'} + \underline{v}_{\Sigma/F'}, \quad \underline{a}_\Sigma = \underline{a}_{O'} + \underline{a}_{\Sigma/F'}$$

ΝΟΜΟΙ ΝΕΥΤΩΝΑ ΚΑΙ EULER



$$\underline{r}_G = \frac{1}{m}\sum_{i=1}^{N} m_i \underline{r}_i, \quad m = \sum_{i=1}^{N} m_i$$

$$\underline{F} = m\ddot{\underline{r}}_G, \quad \underline{M}_P = \sum_{i=1}^{N} \underline{\rho}_i \times m_i \ddot{\underline{r}}_i$$

ΑΡΧΕΣ ΩΣΗΣ ΚΑΙ ΟΡΜΗΣ

$$\underline{L} = \sum_{i=1}^{N} m_i \underline{v}_i = m\underline{v}_G, \quad \underline{F} = \dot{\underline{L}} = m\underline{a}_G$$

$$\hat{\underline{F}} = \int_{t_1}^{t_2} \underline{F}dt = \Delta\underline{L}$$

$$\underline{H}_P = \sum_{i=1}^{N} \underline{\rho}_i \times \left(m_i \underline{v}_i\right), \quad \overline{\underline{H}}_P = \sum_{i=1}^{N} \underline{\rho}_i \times \left(m_i \dot{\underline{\rho}}_i\right)$$

$$\underline{H}_P = \underline{H}_G + \underline{\rho}_G \times \underline{L}, \quad \underline{H}_P = \overline{\underline{H}}_P + \underline{\rho}_G \times \left(m\underline{v}_P\right)$$

$$\underline{M}_P = \dot{\underline{H}}_P + \underline{v}_P \times \left(m\underline{v}_G\right) = \dot{\overline{\underline{H}}}_P + \underline{\rho}_G \times \left(m\ddot{\underline{r}}_P\right)$$

$$\underline{M}_G = \dot{\underline{H}}_G, \quad \hat{\underline{M}}_G = \Delta\underline{H}_G$$

$$\hat{\underline{M}}_P = \int_{t_1}^{t_2} \underline{M}_P dt = \Delta\underline{H}_P + \int_{t_1}^{t_2} \underline{v}_P \times m\underline{v}_G dt$$

ΑΡΧΕΣ ΕΡΓΟΥ ΚΑΙ ΕΝΕΡΓΕΙΑΣ

$$T = \frac{1}{2}mv^2, \quad V_E = \frac{1}{2}kx^2, \quad V_B = mgh$$

$$\underline{F}_C = -\nabla V, \quad E = T + V$$

$$\Delta W = \Delta W_c + \Delta W_{nc} = \Delta T$$

$$\Delta W_{nc} = \int_{r_1}^{r_2} \underline{F}_{nc} \cdot d\underline{r} = E_2 - E_1$$

ΚΡΟΥΣΗ ΥΛΙΚΩΝ ΣΗΜΕΙΩΝ

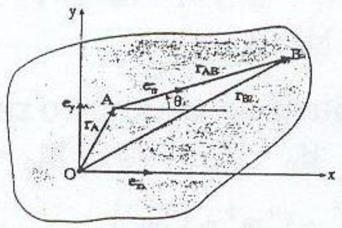
$$e = -\frac{u_2 - u_1}{v_2 - v_1} = \frac{\hat{F}_2}{\hat{F}_1}$$

ΠΕΡΙΣΤΡΟΦΗ ΓΥΡΩ ΑΠΟ ΣΤΑΘΕΡΟ ΑΞΟΝΑ

$$\omega = \dot{\theta}, \quad \alpha = \dot{\omega} = \ddot{\theta}, \quad \underline{\omega} = \omega\underline{e}_z, \quad \underline{\alpha} = \alpha\underline{e}_z$$

$$\underline{v}_\Sigma = \underline{\omega} \times \underline{r}, \quad \underline{a}_\Sigma = \underline{\alpha} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r})$$

ΓΕΝΙΚΗ ΕΠΙΠΕΔΗ ΚΙΝΗΣΗ



$$\omega = \dot{\theta}, \quad \alpha = \dot{\omega} = \ddot{\theta}$$

$$\underline{v}_B = \underline{v}_A + \left(\omega\underline{e}_z\right) \times \underline{r}_{AB}$$

$$\underline{a}_B = \underline{a}_A + \left(\alpha\underline{e}_z\right) \times \underline{r}_{AB} - \omega^2 \underline{r}_{AB}$$

ΠΕΡΙΣΤΡΟΦΗ ΓΥΡΩ ΑΠΟ ΣΤΑΘΕΡΟ ΣΗΜΕΙΟ

$$\underline{v} = \underline{\omega} \times \underline{r}, \quad \underline{a} = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}\right), \quad \underline{\alpha} = \dot{\underline{\omega}}$$

ΠΑΡΑΓΩΓΟΣ ΔΙΑΝΥΣΜΑΤΟΣ

$$\left(\dot{\underline{q}}\right)_F = \left(\dot{\underline{q}}\right)_f + \underline{\omega}_{f/F} \times \underline{q}$$

$$\underline{\omega}_{f'/F} = \underline{\omega}_{f'/f} + \underline{\omega}_{f/F}$$

ΓΕΝΙΚΗ ΧΩΡΙΚΗ ΚΙΝΗΣΗ

$$\underline{v}_B = \underline{v}_A + \underline{\omega} \times \underline{r}_{AB}$$

$$\underline{a}_B = \underline{a}_A + \underline{\alpha} \times \underline{r}_{AB} + \underline{\omega} \times \left(\underline{\omega} \times \underline{r}_{AB}\right)$$

ΣΧΕΤΙΚΗ ΚΙΝΗΣΗ

$$\underline{v}_{\Sigma/F} = \underline{v}_{\Sigma'/F} + \underline{v}_{\Sigma/f}$$

$$\underline{a}_{\Sigma/F} = \underline{a}_{\Sigma'/F} + \underline{a}_{\Sigma/f} + 2\underline{\omega}_{f/F} \times \underline{v}_{\Sigma/f}$$

ΚΙΝΗΤΙΚΗ ΣΤΕΡΕΩΝ ΣΩΜΑΤΩΝ

$$\underline{r}_G = \frac{1}{m}\int_m \underline{r}\,dm, \quad m = \int_m dm$$

$$\underline{L} = m\underline{v}_G, \quad \underline{F} = \dot{\underline{L}} = m\underline{a}_G$$

$$\overline{\underline{H}}_P = I_P\underline{\omega}, \quad I_P = \begin{pmatrix} I^P_{xx} & I^P_{xy} & I^P_{xz} \\ I^P_{yx} & I^P_{yy} & I^P_{yz} \\ I^P_{zx} & I^P_{zy} & I^P_{zz} \end{pmatrix}$$

$$\underline{H}_G = \overline{\underline{H}}_G = I_G\underline{\omega}$$

$$\underline{M}_G = \dot{\underline{H}}_G = \left(\dot{\underline{H}}_G\right)_f + \underline{\omega}\times\underline{H}_G \Rightarrow$$

$$M_x = I_{xx}\dot{\omega}_x + I_{xy}\left(\dot{\omega}_y - \omega_z\omega_x\right) + I_{zx}\left(\dot{\omega}_z + \omega_x\omega_y\right) + \left(I_{zz} - I_{yy}\right)\omega_z\omega_y + I_{yz}\left(\omega_y^2 - \omega_z^2\right),$$

$$M_y = I_{yy}\dot{\omega}_y + I_{yz}\left(\dot{\omega}_z - \omega_x\omega_y\right) + I_{xy}\left(\dot{\omega}_x + \omega_y\omega_z\right) + \left(I_{xx} - I_{zz}\right)\omega_x\omega_z + I_{zx}\left(\omega_z^2 - \omega_x^2\right),$$

$$M_z = I_{zz}\dot{\omega}_z + I_{zx}\left(\dot{\omega}_x - \omega_y\omega_z\right) + I_{yz}\left(\dot{\omega}_y + \omega_z\omega_x\right) + \left(I_{yy} - I_{xx}\right)\omega_y\omega_x + I_{xy}\left(\omega_x^2 - \omega_y^2\right)$$

$$T = T_G + T_R, \quad T_G = \frac{1}{2}mv_G^2$$

$$T_R = \frac{1}{2}\left(I^G_{xx}\omega_x^2 + I^G_{yy}\omega_y^2 + I^G_{zz}\omega_z^2\right) + I^G_{xy}\omega_x\omega_y + I^G_{yz}\omega_y\omega_z + I^G_{zx}\omega_z\omega_x$$

$$P = \underline{F}\cdot\underline{v}_G + \underline{M}_G\cdot\underline{\omega}$$

ΠΕΡΙΣΤΡΟΦΗ ΩΣ ΠΡΟΣ ΑΚΙΝΗΤΟ ΣΗΜΕΙΟ

$$\underline{M}_O = \dot{\underline{H}}_O, \quad \underline{H}_O = \overline{\underline{H}}_O = I_O\underline{\omega}, \quad \hat{\underline{M}}_O = \Delta\underline{H}_O$$

$$T = \frac{1}{2}\left(I^O_{xx}\omega_x^2 + I^O_{yy}\omega_y^2 + I^O_{zz}\omega_z^2\right) + I^O_{xy}\omega_x\omega_y + I^O_{yz}\omega_y\omega_z + I^O_{zx}\omega_z\omega_x$$

ΕΠΙΠΕΔΗ ΚΙΝΗΣΗ

$$\underline{\omega} = \omega\underline{e}_z, \quad \underline{H}_G = \left(I^G_{xz}\underline{e}_x + I^G_{yz}\underline{e}_y + I^G_{zz}\underline{e}_z\right)\omega$$

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I^G_{zz}\omega^2$$

ΠΕΡΙΣΤΡΟΦΗ ΩΣ ΠΡΟΣ ΑΞΟΝΑ

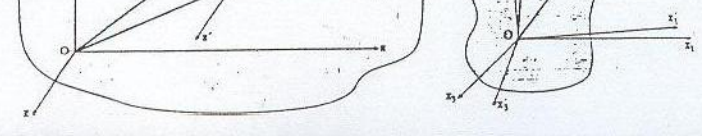
$$M = I\alpha, \quad \hat{M} = I\Delta\omega, \quad T = \frac{1}{2}I\omega^2$$

ΜΕΤΑΦΟΡΙΚΗ ΚΙΝΗΣΗ

$$\underline{\omega} = \underline{0} \Rightarrow \underline{M}_G = \underline{0}$$

$$\underline{M}_P = \underline{\rho}_G \times \left(m\underline{a}_G\right)$$

ΘΕΩΡΗΜΑ ΠΑΡΑΛΛΗΛΩΝ ΑΞΟΝΩΝ



$$I_O = I_G + m\begin{pmatrix} y_G^2 + z_G^2 & -x_Gy_G & -x_Gz_G \\ -y_Gx_G & z_G^2 + x_G^2 & -y_Gz_G \\ -z_Gx_G & -z_Gy_G & x_G^2 + y_G^2 \end{pmatrix}$$

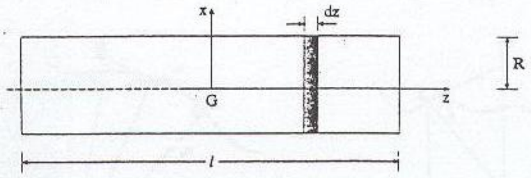
ΠΕΡΙΣΤΡΟΦΗ ΑΞΟΝΩΝ

$$I'_O = AI_OA^T, \quad A = \left[a_{ij}\right], \quad a_{ij} = \underline{e'}_i\cdot\underline{e}_j$$

ΑΚΤΙΝΑ ΑΔΡΑΝΕΙΑΣ $r_G = \sqrt{I/m}$

ΜΑΖΙΚΕΣ ΡΟΠΕΣ ΑΔΡΑΝΕΙΑΣ

Κύλινδρος



$$I^G_{xx} = I^G_{yy} = \frac{1}{4}mR^2 + \frac{1}{12}mL^2, \quad I^G_{zz} = \frac{1}{2}mR^2$$

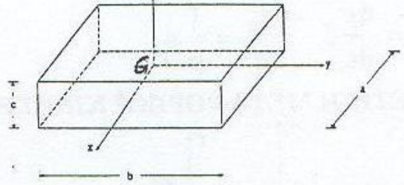
Ράβδος

$$I^G_{xx} = I^G_{yy} = \frac{1}{12}mL^2, \quad I^G_{zz} \cong 0$$

Δίσκος

$$I^G_{zz} = \frac{1}{2}mR^2 = 2I^G_{xx} = 2I^G_{yy}$$

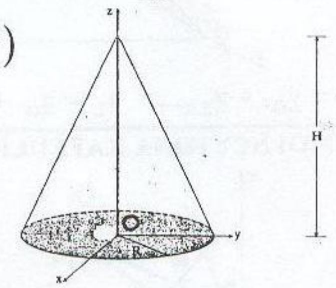
Ορθογώνιο



$$I^G_{xx} = \frac{m}{12}\left(b^2 + c^2\right), \quad I^G_{yy} = \frac{m}{12}\left(c^2 + a^2\right),$$

$$I^G_{zz} = \frac{m}{12}\left(a^2 + b^2\right)$$

Κώνος



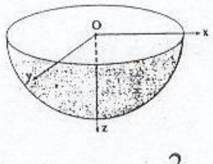
$$I^O_{xx} = I^O_{yy} = \frac{m}{20}\left(3R^2 + 2H^2\right),$$

$$I^O_{zz} = \frac{3}{10}mR^2, \quad \underline{r}_G = \frac{H}{4}\underline{e}_z$$

Σφαίρα

$$I^G_{xx} = I^G_{yy} = I^G_{zz} = \frac{2}{5}mR^2$$

Ημισφαίριο



$$I^O_{xx} = I^O_{yy} = I^O_{zz} = \frac{2}{5}mR^2, \quad \underline{r}_G = \frac{3}{8}R\underline{e}_z$$